

**Trigonometria/Trigonometri/Trigonometry**

$$\sin^2 x + \cos^2 x = 1 \quad \tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{1}{\tan x}$$

$$\sin x = \pm \sqrt{1 - \cos^2 x} = \pm \frac{\tan x}{\sqrt{1 + \tan^2 x}} \quad \cos x = \pm \sqrt{1 - \sin^2 x} = \pm \frac{1}{\sqrt{1 + \tan^2 x}}$$

$$\sin x = -\sin(-x) = \cos\left(\frac{\pi}{2} - x\right) = \sin(\pi - x) = \sin(x + n2\pi)$$

$$\cos x = \cos(-x) = \sin\left(\frac{\pi}{2} - x\right) = -\cos(\pi - x) = \cos(x + n2\pi)$$

$$\tan x = -\tan(-x) = \cot\left(\frac{\pi}{2} - x\right) = -\tan(\pi - x) = \tan(x + n\pi)$$

$$\sin 2x = 2 \sin x \cos x \quad \cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$$

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma} \quad a^2 = b^2 + c^2 - 2bc \cos \alpha$$

**Vektorit/Vektorer/Vectors** ( $k = \text{vakio/konstant/constant}$ )

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}, \text{ missä/där/where } \vec{a} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos(\vec{a}, \vec{b}) = a_x b_x + a_y b_y + a_z b_z \quad a_b = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \quad \vec{a}_b = \frac{\vec{a} \cdot \vec{b}}{\vec{b} \cdot \vec{b}} \vec{b}$$

**Derivointi/Derivering/Differentiation** ( $k = \text{vakio/konstant/constant}$ )

$$Dk = 0 \quad Dkf = k Df \quad D(f + g) = Df + Dg$$

$$Dfg = f Dg + g Df \quad D\frac{f}{g} = \frac{g Df - f Dg}{g^2} \quad Dg(f(x)) = g'(f(x))f'(x)$$

$$(f^{-1})'(y_0) = \frac{1}{f'(x_0)}, \text{ missä/där/where } y_0 = f(x_0)$$

$$Dx^n = nx^{n-1} \quad Df^n = n f^{n-1} Df$$

$$D \sin x = \cos x \quad D \cos x = -\sin x \quad D \tan x = \frac{1}{\cos^2 x} = 1 + \tan^2 x$$

$$De^x = e^x \quad Da^x = a^x \ln a \quad (a > 0) \quad D \ln |x| = \frac{1}{x} \quad D \log_a |x| = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$$

**Integrointi/Integrering/Integration** ( $k = \text{vakio/konstant/constant}$ )

$$\int kf \, dx = k \int f \, dx \quad \int (f + g) \, dx = \int f \, dx + \int g \, dx \quad \int f' g \, dx = fg - \int g' f \, dx$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1) \quad \int \frac{1}{x} \, dx = \ln|x| + C$$

$$\int f' f^n \, dx = \frac{f^{n+1}}{n+1} + C \quad (n \neq -1) \quad \int \frac{f'}{f} \, dx = \ln|f| + C$$

$$\int \sin x \, dx = -\cos x + C \quad \int \cos x \, dx = \sin x + C \quad \int \tan x \, dx = -\ln|\cos x| + C$$

$$\int e^x \, dx = e^x + C \quad \int f' e^f \, dx = e^f + C \quad \int a^x \, dx = \frac{a^x}{\ln a} + C \quad (a > 0, a \neq 1)$$

$$\int f'(x)g(f(x)) \, dx = G(f(x)) + C, \text{ missä/där/where } DG = g$$