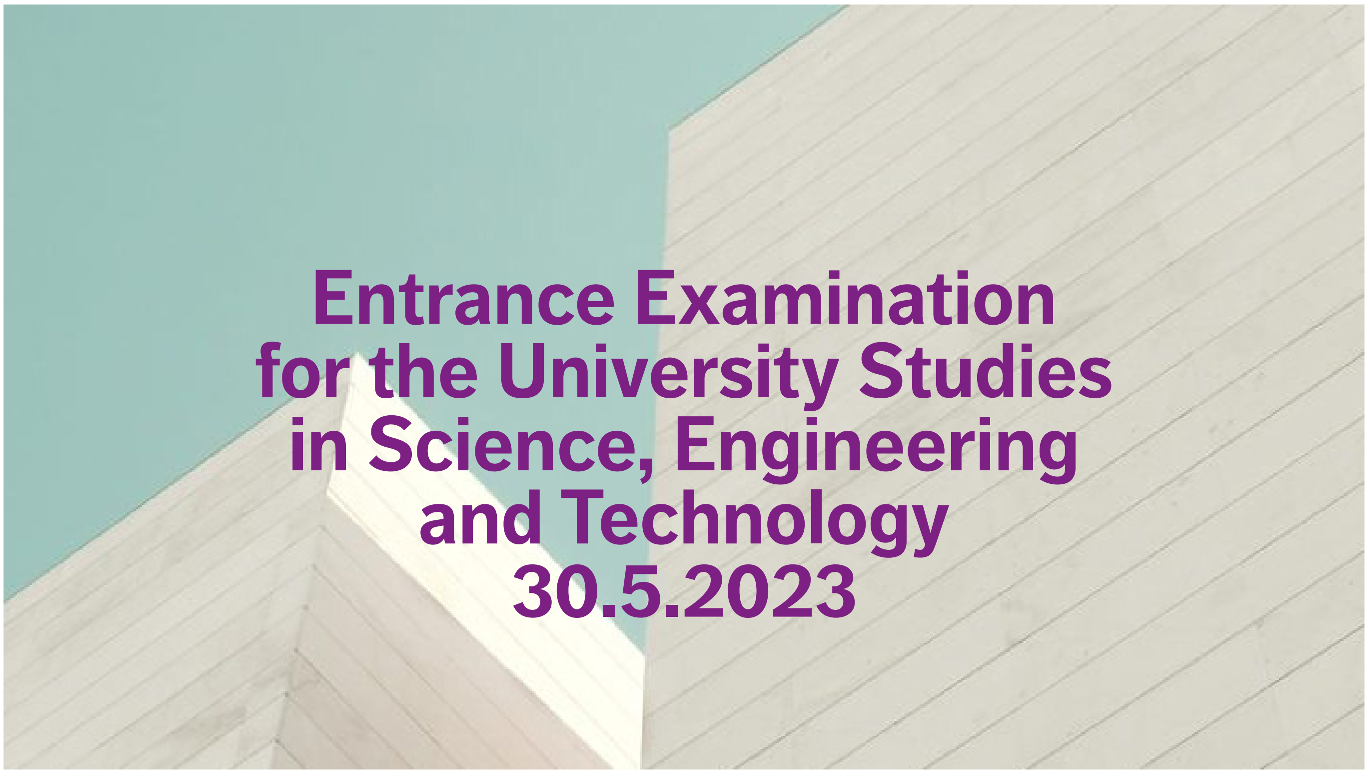
DIPLOMI-INSINÖÖRI- JA ARKKITEHTIKOULUTUKSEN YHTEISVALINTA
DIPLOMINGENJÖRS- OCH ARKITEKTUTBILDNINGENS GEMENSAMMA ANTAGNING



Entrance Examination for the University Studies in Science, Engineering and Technology 30.5.2023

QUESTIONS

INSTRUCTIONS

The entrance examination has four sections: mathematics, physics, chemistry and problem solving. The mathematics section is mandatory for everyone. Please answer all the questions in the mathematics section. In addition, please answer three (3) more questions from the optional sections. If you answer to more than three optional questions, only those three questions that give the lowest scores are considered.

Please write all your answers in the answer booklet on the question-specific answer sheet. Only those answers will be considered. After the examination, you may take the question booklet with you.

QUESTION BOOKLET
Do not answer here.

Justify your answers in all mathematics questions.

Mathematics | Question 1.
Report the answers in exact values.

a) Solve the equation $\pi x + 3 = \sqrt{2} + 4x$. (1 p.)

b) Solve the equation $(x - 2)(x - 3) = 6$. (1 p.)

c) Solve the inequality $x^2 - 4 > 0$. (1 p.)

d) Calculate $\sum_{n=1}^{4}(3n + 2)$. (1 p.)

e) Calculate the dot product of vectors $\overline{v} = \overline{i} + 2\overline{j} + 3\overline{k}$ and $\overline{w} = -3\overline{i} + \overline{j} - 2\overline{k}$. (1 p.)

f) Which are the solutions of equation $\cos t = \sin t$, when $0 \leq t \leq 2\pi$? (1 p.)

Mathematics | Question 2.

a) A student hopes that a day of his holiday is not rainy (probability 60 %), that he could meet friends (probability 70 %), and that he wouldn't miss the return train (probability 90 %). The events are mutually independent. The day of the holiday is **excellent**, if all wishes come true and **good**, if two of the wishes come true.

What is the probability that the day of the holiday is excellent? And with what probability good? (2 p.)

b) Freighter Kalle and his son Tomi transport two truckloads of paper towels from the paper mill to the customer. Because the paper is light, Kalle and Tomi load up their trucks by hand as full as possible. The trucks are the same size and one loader does not disturb the other.

With his long experience, Kalle can fully load his own car in two hours. When co-operating, they can fully load both trucks in 144 minutes. How long it would take Tomi to fully load his truck alone? (4 p.)

Mathematics | Question 3.

a) Show that the function $f(x) = \sin^2 x - 2x + 1$ has exactly one zero, when $0 \leq x \leq 1$. (3 p.)

b) Compare the integrals

$$\int_{-2}^{-1} \left(|x| + x^3\right) dx \quad \text{and} \quad \int_{1}^{2} \left(|x| + x^3\right) dx.$$

Is one of them greater? If yes, which? (3 p.)

Physics │Question 1.

Answer the sub-questions 1–4. In multiple-choice questions, select one answer (A–D). Correct answer: 1 p. Wrong answer, no answer, or multiple answers: 0 p.

The International Space Station (ISS) orbits the Earth in a circular path with a constant speed at an altitude of 420 km from the Earth's surface. The Earth attracts the space station with a 3.8 MN force. The mass of the station is 444 000 kg. The mass of the Earth is $5.97 \cdot 10^{24}$ kg, the radius of the Earth is 6371 km and the gravitational constant is $6.67 \cdot 10^{-11}$ m^3/kgs^2.

1. Which of the following options describes the force that the space station exerts on the Earth. (1 p.)

   A. The absolute value of the force is zero, i.e., the space station does not exert a force on the Earth.
   B. The absolute value of the force is smaller than 3.8 MN, but not zero.
   C. The absolute value of the force is 3.8 MN.
   D. The absolute value of the force is greater than 3.8 MN.

2. Which of the following options is correct? When ISS orbits the Earth, (1 p.)

   A. its acceleration is zero.
   B. the direction of its acceleration points towards the center of the Earth.
   C. the direction of its acceleration is parallel to the direction of ISS's velocity.
   D. the direction of its acceleration is antiparallel to the direction of ISS's velocity.

3. The astronaut feels weightless in the ISS. Which of the following statements best describes the reason for the weightlessness? (1 p.)

   A. Gravity in space is negligible.
   B. The astronaut falls closer to the center of the Earth all the time.
   C. The ISS does not exert a supporting force to the astronaut.
   D. Mass is not relevant in space.

4. How long does it take for the ISS to orbit the Earth for one revolution? Please give arguments to support your answer. (3 p.)

Physics | Question 2.

Based on the reference material, answer the sub-questions 1–4. In multiple-choice questions, select one answer (A–D). Correct answer: 1 p. Wrong answer, no answer, or multiple answers: 0 p.

In sub-questions 1 and 2, the term "recoverable energy" refers to the energy produced by exciting electrons from the valence band to the conduction band in silicon. You should estimate only the effect of the two loss mechanisms presented in the reference material.

1. How much is it possible to recover from the energy contained in green light (λ = 550 nm)? (1 p.)

   A. about 0 %.
   B. about 25 %.
   C. about 50 %.
   D. about 100 %.

2. A cell absorbs 1.0 J of blue light and 1.0 J of red light. Which of the following statements is true? (1 p.)

   A. 1.0 J of red light produces as much recoverable energy as 1.0 J of blue light.
   B. 1.0 J of red light produces more recoverable energy than 1.0 J of blue light.
   C. 1.0 J of red light produces less recoverable energy than 1.0 J of blue light.
   D. It is impossible to determine if the statements A-C are true based on the information given.

3. The solar panel described in the reference material is connected in series with a device using direct current as shown in Figure 1. The measured voltage output of the solar panel is 44 V. The mutual dependence between the voltage and current produced by the panel is shown in Figure 2 of the reference material. There are no other power losses in the circuit except the power used by the device. How much power the device is using? (1 p.)

   A. 44 W
   B. 110 W
   C. 120 W
   D. 160 W

4. The solar panel described in the reference material is connected in series with a device using direct current as shown in Figure 1. The resistance of the device is 10.0 Ω. The mutual dependence between the voltage and current produced by the panel is shown in Figure 2 of the reference material. There are no other power losses in the circuit except the power used by the device. How much power the device is using? If you base your answer on a figure in the reference material, copy it in your answer, as well. (3 p.)

Physics│Question 2. Reference material: Semiconductor solar cells

A solar cell produces electrical energy directly from sunlight composed of energy packets called photons. When an active material in the solar cell absorbs a photon, the energy of the photon can excite an electron to a higher energy state.

The most common commercially used active material is monocrystalline silicon. Silicon is a semiconductor, which means that the forbidden energy zone (i.e. energy gap E_g) between its valence and conduction bands is rather small. The energy gap of silicon is 1.1 eV, which corresponds to a photon with wavelength of 1100 nm. An electronvolt (eV) is a unit of energy: 1 eV = $1.602 \cdot 10^{-19}$ J.

When a photon hits the cell with an energy of at least the size of the energy gap, it can excite an electron from the valence band to the conduction band. This creates a voltage between the poles of the solar cell. The energy gap also sets an upper limit to the amount of recoverable energy: if the energy of the photon is larger than the energy gap, the amount of energy exceeding the gap is lost.

These properties lead to two major loss mechanisms:

- The photons with an energy smaller than the energy gap cannot be utilized at all.
- For the photons with an energy larger than the energy gap, the energy can be recovered only up to the amount corresponding to the energy gap, and the exceeding part of the energy is lost.

In reality there are several other loss mechanisms in solar cells, but the two mechanisms mentioned above cause already over 50 % power loss in solar cells made of a single material. This is due to the fact that sunlight hitting the Earth consists of a continuum spectrum of photons with different wavelengths: visible light from red (620 nm - 750 nm) to purple (380 nm - 450 nm), and in addition photons of infrared and ultraviolet regions. When considering the above mentioned loss mechanisms, the central role is played by the energy of the photon with a certain wavelength. The energy E and frequency of the photon are directly connected: $E = hf$. Here h = $6.626 \cdot 10^{-34}$ Js is Planck's constant.

In practical applications of solar energy, solar panels with several cells connected in series to increase the voltage are applied. Figure 1 shows a circuit, where a solar panel is connected in series with a device using solar power. The panel produces voltage U and current I in the circuit, and the produced power is consumed in the device (a resistor with resistance R). The manufacturer of this solar panel has carried out a measurement of the dependence of the current on the voltage in standard conditions (light intensity 1000 W/m^2, light spectrum AM1.5G and panel temperature 25 °C) producing the graph presented in Figure 2.

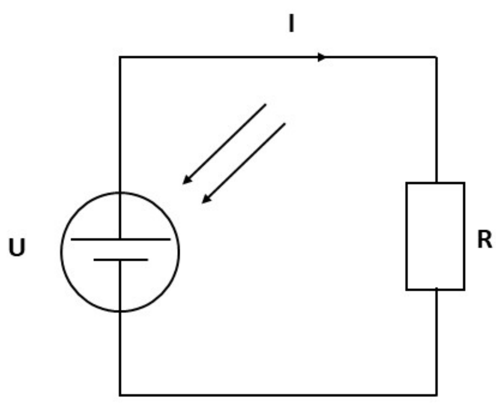


Figure 1. Electric circuit. Solar panel produces voltage U and current I. The device (resistance R) consumes the produced solar energy.

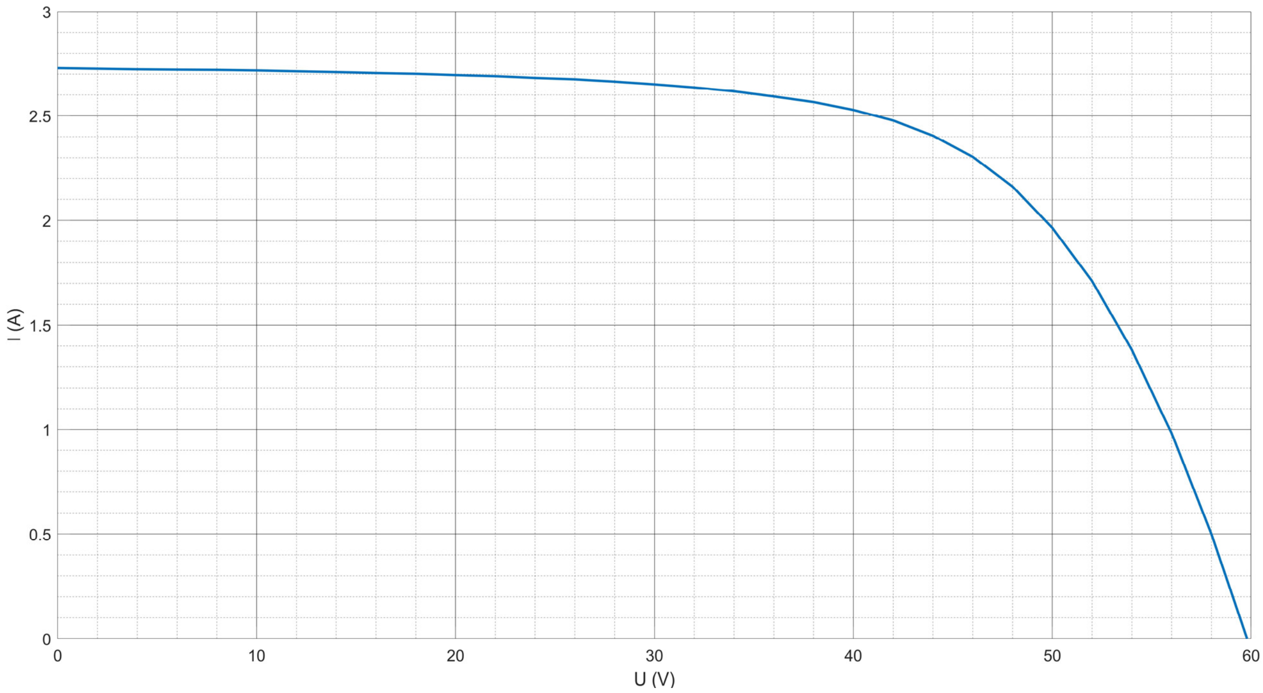


Figure 2. Relationship between the voltage *U* and current *I* produced by a solar cell measured in standard conditions.

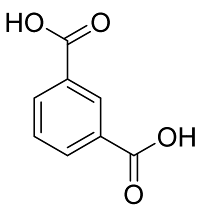
Chemistry | Question 1.

Answer the sub-questions 1–4. In multiple-choice questions, select one answer (A–D). Correct answer: 1 p. Wrong answer, no answer, or multiple answers: 0 p.

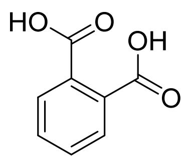
You can use the attached periodic table of the elements when answering the questions. Please note that the decimal separator used in the periodic table of elements is comma (,).

1. Which of the following statements is <u>incorrect?</u> (1 p.)
   A. Selenium (Se, *Z* = 34) obtains the electron configuration of a noble gas by receiving two electrons.
   B. The charge of a rubidium cation (Rb, *Z* = 37) is 1+.
   C. The electronegativity of phosphorus (P, *Z* = 15) is lower than the electronegativity of sulfur (S, *Z* = 16).
   D. The valence electrons of iodine (I, *Z* = 53) are located in the electron shell 4.

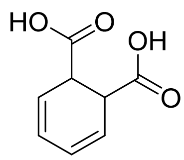
2. Phthalic acid is a diprotic aromatic carboxylic acid. The carboxylic acid groups are in the *ortho* position, meaning that they are bonded to benzene ring carbon atoms 1 and 2. Which of the structural formulas (A-D) represents phthalic acid? (1 p.)



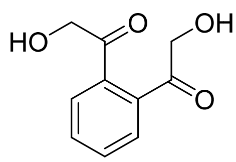
A



B

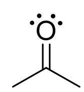


C



D

3. Which of the following statements is <u>correct</u> (1 p.)
   A. A hydrogen bond is a polar covalent bond between a hydrogen atom, and an atom that is more electronegative than hydrogen.
   B. A hydrogen bond is formed between two hydrogen atoms that are bonded to an atom that is more electronegative than hydrogen.
   C. Two molecules are always required to form a hydrogen bond.
   D. Hydrogen bonds can be formed between acetone molecules and water molecules.



acetone

4. Draw the structural formulas of all compounds with a molecular formula C_4H_8. Conformational isomers do not need to be considered. (3 p.)

Chemistry │Question 2.

Based on the reference material, answer the sub-questions 1–4. In multiple-choice questions, select one answer (A–D). Correct answer: 1 p. Wrong answer, no answer, or multiple answers: 0 p.

You can use the attached periodic table of the elements when answering the questions. Please note that the decimal separator used in the periodic table of elements is comma (,).

1. Precipitate A in Scheme 1 is (1 p.)
   A. Graphite
   B. Cobalt hydroxide
   C. Lithium hydroxide
   D. Sodium hydroxide

2. Examine the reaction where lithium cobalt oxide is dissolved in sulfuric acid solution that contains hydrogen peroxide. Which of the following statements is correct? (1 p.)
   A. Cobalt is reduced and oxygen is oxidised.
   B. Lithium is oxidised and oxygen is reduced.
   C. Lithium and cobalt are oxidised.
   D. Cobalt and oxygen are reduced, and lithium is oxidised.

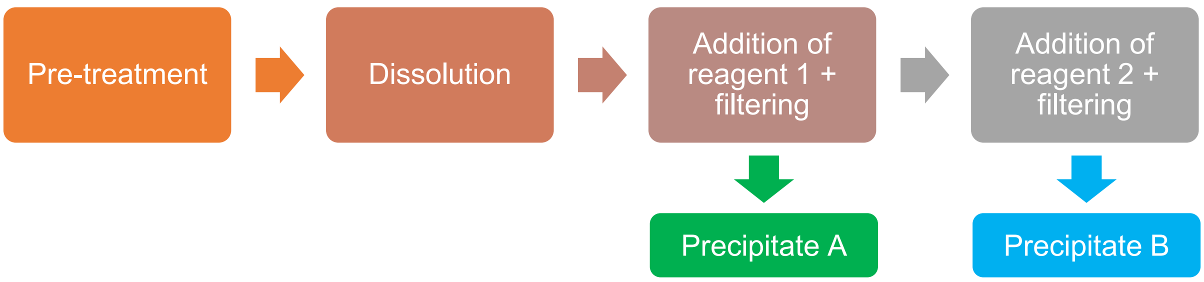
3. Why there are two pK_A values given for the oxalic acid in the reference material? (1 p.)
   A. Since oxalic acid can act as both an acid and a reducing agent.
   B. Since oxalic acid forms a water-soluble salt with lithium, and a precipitate with cobalt.
   C. Since oxalic acid is a weak diprotic acid.
   D. Since oxalic acid is a strong diprotic acid.

4. A sample of lithium-ion battery waste was treated according to Scheme 1. With this method, 96% of lithium can be recovered. How many grams of lithium cobalt oxide (M = 97.87 g/mol) did the sample originally contain, if 7.54 grams of lithium carbonate (M = 73.89 g/mol) precipitated? Dissolution was carried out by using sulfuric acid solution that contains hydrogen peroxide. Justify your answer. (3 p.)

Chemistry | Question 2. Reference material: Recovery of battery metals

The European Commission updates the list of critical raw materials (CRM) every few years. The list includes several metals that are essential for example in hybrid and electric cars, solar panels, and electronic devices. The cathode material in the commonly used lithium-ion batteries is typically lithium cobalt oxide ($LiCoO_2$), while the anode material is often graphite or another carbon-based material. The essential components in lithium-ion batteries (lithium, cobalt, graphite) are included in the list of critical raw materials. Recycling of battery materials is, hence, essential to guaranteeing the availability of battery metals in the future. The challenge lies in the fact that the battery waste contains several different materials, and a multistep process is required to recover all the different metals.

A simplified process for treatment of used lithium-ion batteries is shown in Scheme 1. The scheme describes specifically the recovery of lithium and cobalt. The process includes pre-treatment, dissolution, and separation steps. Graphite and metals, other than those in electrode materials, are separated in the pre-treatment step. The dissolution in this case refers to the treatment of the cathode material with an acid solution, where metals form water-soluble salts. Inorganic acids, such as H_2SO_4 (pK_{a1} = −2.8; pK_{a2} = 1.99) and HCl (pK_{a1} = −5.9) are the most common acids for this purpose. Hydrogen chloride (HCl) is an excellent acid for dissolution of metals, but the poisonous chlorine gas formed in the reaction is problematic in many ways.



Scheme 1: A simplified process for recovery of lithium and cobalt.

When sulfuric acid solution is used for dissolution of the cathode material, hydrogen peroxide that acts as a reducing agent, is often added in the reaction mixture. The dissolution of lithium cobalt oxide in sulfuric acid solution that contains hydrogen peroxide can be described according to the reaction equation below:

$$2\ LiCoO_2(s) + 6\ H^+(aq) + H_2O_2(aq) \rightarrow 2\ Co^{2+}(aq) + O_2(g) + 2\ Li^+(aq) + 4\ H_2O(l).$$

Since both lithium and cobalt are dissolved in the acid solution, different methods are required for their separation and recovery. In the process described in Scheme 1, cobalt and lithium are separated by precipitating them from the solution. First, the solution formed in the dissolution step is made basic (pH 11) by adding NaOH-solution, which results in the precipitation of cobalt as cobalt hydroxide ($Co(OH)_2$). After filtering off the precipitate, saturated Na_2CO_3 solution is added and lithium, in turn, precipitates as lithium carbonate (Li_2CO_3).

Organic acids, such as oxalic acid (pK_{a1} = 2.27; pK_{a2} = 4.28), can also be utilised in the dissolution of cathode material. Here, the overall reaction can be described according to the reaction equation below:

$$2\ LiCoO_2(s) + 4\ H_2C_2O_4(aq) \rightarrow Li_2C_2O_4(aq) + 2\ CoC_2O_4(s) + 4\ H_2O(l) + 2\ CO_2(g).$$

Lithium oxalate that is formed in the reaction is soluble in the acid solution, whereas cobalt oxalate is precipitated from the solution. In this case, the addition of hydrogen peroxide has not been shown to have any significant effect on dissolution efficiency, and oxalic acid can thus be assumed to act as a reducing agent in the reaction.

Neither of the processes described above are optimal, and it is obvious that the recovery processes still require a lot of development work. For this development, solid proficiency in chemistry plays a key role.

Periodic table of elements

järjestysluku / atomnummer / atomic number
kemiallinen merkki / kemiskt tecken / chemical symbol
atomimassa / atommassa /atomic mass

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 **H** 1,008																	2 **He** 4,003
2	3 **Li** 6,941	4 **Be** 9,012											5 **B** 10,81	6 **C** 12,01	7 **N** 14,01	8 **O** 16,00	9 **F** 19,00	10 **Ne** 20,18
3	11 **Na** 22,99	12 **Mg** 24,31											13 **Al** 26,98	14 **Si** 28,09	15 **P** 30,97	16 **S** 32,07	17 **Cl** 35,45	18 **Ar** 39,95
4	19 **K** 39,10	20 **Ca** 40,08	21 **Sc** 44,96	22 **Ti** 47,87	23 **V** 50,94	24 **Cr** 52,00	25 **Mn** 54,94	26 **Fe** 55,85	27 **Co** 58,93	28 **Ni** 58,69	29 **Cu** 63,55	30 **Zn** 65,38	31 **Ga** 69,72	32 **Ge** 72,63	33 **As** 74,92	34 **Se** 78,96	35 **Br** 79,90	36 **Kr** 83,80
5	37 **Rb** 85,47	38 **Sr** 87,62	39 **Y** 88,91	40 **Zr** 91,22	41 **Nb** 92,91	42 **Mo** 95,94	43 **Tc** (98)	44 **Ru** 101,07	45 **Rh** 102,91	46 **Pd** 106,42	47 **Ag** 107,87	48 **Cd** 112,41	49 **In** 114,82	50 **Sn** 118,71	51 **Sb** 121,76	52 **Te** 127,60	53 **I** 126,90	54 **Xe** 131,29
6	55 **Cs** 132,91	56 **Ba** 137,33	57-71	72 **Hf** 178,49	73 **Ta** 180,95	74 **W** 183,84	75 **Re** 186,21	76 **Os** 190,23	77 **Ir** 192,22	78 **Pt** 195,08	79 **Au** 196,97	80 **Hg** 200,59	81 **Tl** 204,38	82 **Pb** 207,2	83 **Bi** 208,98	84 **Po**	85 **At**	86 **Rn**
7	87 **Fr**	86 **Ra**	89-103	104 **Rf**	105 **Db**	106 **Sg**	107 **Bh**	108 **Hs**	109 **Mt**	110 **Ds**	111 **Rg**	112 **Cn**	113 **Nh**	114 **Fl**	115 **Mc**	116 **Lv**	117 **Ts**	118 **Og**

Lantanoidit/ Lantanoider/ Lanthanides	57 **La** 138,91	58 **Ce** 140,12	59 **Pr** 140,91	60 **Nd** 144,24	61 **Pm**	62 **Sm** 150,36	63 **Eu** 151,96	64 **Gd** 157,25	65 **Tb** 158,93	66 **Dy** 162,50	67 **Ho** 164,93	68 **Er** 167,26	69 **Tm** 168,93	70 **Yb** 173,05	71 **Lu** 174,97
Aktinoidit/ Aktinoider/ Actinides	89 **Ac**	90 **Th** 232,04	91 **Pa** 231,04	92 **U** 238,03	93 **Np**	94 **Pu**	95 **Am**	96 **Cm**	97 **Bk**	98 **Cf**	99 **Es**	100 **Fm**	101 **Md**	102 **No**	103 **Lr**

Problem Solving │Question 1.

On the next page, there is reference material related to the Finnish passenger car fleet. Answer the sub-questions 1–5 using only the information provided. In the multiple-choice questions, select one answer (A–D). Correct answer: 1 p. Wrong answer, no answer, or multiple answers: 0 p.

Assume that cars using different power sources are driven approximately the same number of kilometers and have approximately the same lifetime, unless a sub-question specifies otherwise. Here, the driving costs of a passenger car consist only of fuel consumption or electricity consumption, and in some sub-questions also a tax on driving power. Other costs such as insurances, maintenance, etc. are not considered.

1. If the trends in the data continue, what will be the most common power source in the passenger car fleet in 2027? (1 p.)

   A. Gasoline
   B. Diesel
   C. Alternative driving powers altogether
   D. Cannot be reasoned using the information provided

2. If the trends in the data continue, including emissions from fuel production, which power source will cause the most CO_2 emissions in 2029? (1 p.)

   A. Gasoline
   B. Diesel
   C. Alternative driving powers altogether
   D. Cannot be reasoned using the information provided

3. Which of the following statements is true based on the data provided? (1 p.)

   A. The majority of CO_2 emissions from the passenger car fleet is produced by diesel cars.
   B. If the average price of natural gas were 30% higher, it would be the most expensive power source per 100 km for the best selling cars in the segment, excluding the tax on driving power.
   C. If the annual mileage of a 1,900 kg electric car is 5,000 km, then the tax on driving power represents less than 35% of the annual driving cost of the electric car.
   D. None of the above options A–C.

4. How much is the minimum annual mileage of a natural gas car with a total mass of 1,950 kg such that its driving costs are less than the driving costs of a gasoline car when the tax on driving power is considered? Use the average prices and reference consumptions in the reference material. (1,5 p.)

5. Let the purchase price of a Tesla Model 3 be €50,000, total mass 2,150 kg and consumption 16.8 kWh/100km. Let the purchase price of a gasoline Volkswagen Golf be €30,000, with a total mass of 1,750 kg and consumption of 5.3 l/100km. Assume that the average prices and taxes remain constants as in the data provided, and that both cars are driven 200 km on average per week. How many years must the Tesla be driven so that its higher purchase price would be repaid by smaller driving costs? Tax on driving power must be included in the calculation. (1,5 p.)

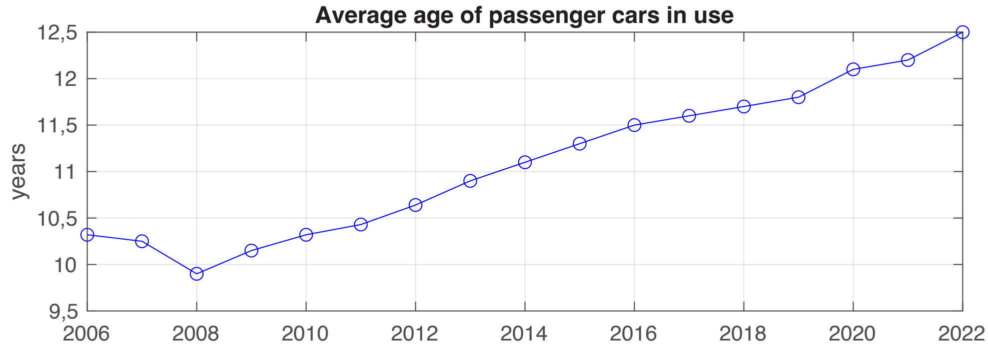


Figure 1. Average age of passenger cars in use.

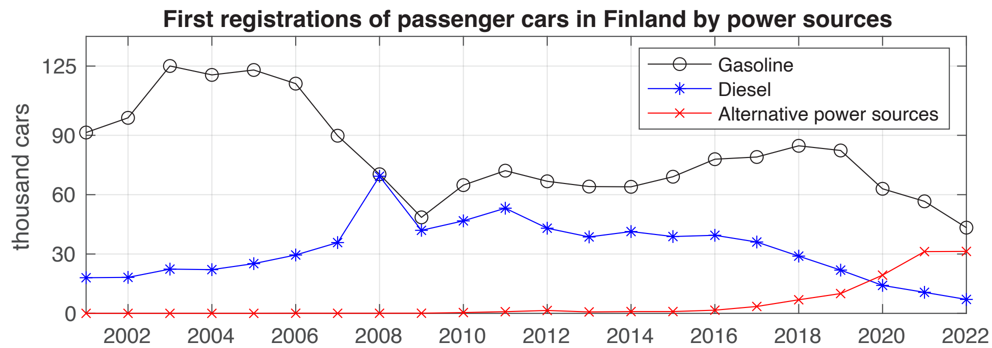


Figure 2. First registrations of passenger cars in Finland by power sources.

Table 1. Average prices of power sources, consumptions, and CO_2 emissions of the best-selling car models.

Power source	Average price 2022-04-01–06-30	Unit	Most sold segment in 2021	Best-selling car models of the segment C in 2021	Reference consumption	Unit / 100 km	CO_2-emission, g/km [3]
Gasoline (E10)	2.35	EUR/l	C	Toyota Corolla, Skoda Octavia, Volkswagen Golf	5.3	l	113
Diesel (B7) [1]	2.31	EUR/l		Skoda Octavia, Volkswagen Golf, Mercedes-Benz A-series	5.5	l	134
Natural gas (CNG) [1]	2.49	EUR/kg		Skoda Octavia, Volkswagen Golf, Skoda Scala	3.9	kg	106
Biogas (CNG) [1]	1.95	EUR/kg		Skoda Octavia, Volkswagen Golf, Skoda Scala	3.9	kg	106
Electricity (C) [1]	24.2	cent/kWh [2]		Tesla Model 3, Volkswagen ID.3, Nissan Leaf	16.8	kWh	0

[1] Cars using these power sources are also subject to a tax on driving power, which depends on the car's mass and power source.

[2] Based on the household price of electricity including transmission charges and taxes.

[3] Excluding emissions from fuel production. According to Fingrid's and Gasum's data, the calculated CO_2 emissions, including the production of fuel and electricity together with losses, for a biogas car are 33.2 g/km and for an electric car are 16.9 g/km.

The tax on driving power is levied on a car in traffic use for each initial 100 kg of total mass as follows:

- Diesel: 5.5 cent/day
- Natural gas and biogas: 3.1 cent/day
- Electricity: 1.5 cent/day

The impact of the tax on driving power on driving costs (€/100km) depends on the number of kilometers driven each year. For example, for a car used in traffic all year round and driven 17,000 km per year, the cost impact is approximately

- Diesel: 1.5–2.2 €/100km
- Natural gas and biogas: 0.9–1.3 €/100km
- Electricity: 0.4–0.6 €/100km

The reference material is based on the information from the Finnish Transport and Communications Agency

Problem Solving | Question 2.

Each of sub-questions 1–6 yields at most one points. *A prime number* is an integer that is greater than 1 and that cannot be expressed as a product of positive integers that are smaller than the prime number itself. For instance, 6 is not a prime number because $6 = 2 \cdot 3$, but 2, 3, 5 and 7 are prime numbers.

1. Among the alternatives below, choose all whose result is a prime number. If your answer contains precisely one error, then you get 0.5 points. Both choosing a wrong alternative and not choosing a right alternative are errors.

   A. $13 \cdot 1$ B. $\frac{2 \cdot 2}{4}$ C. $2 \cdot 7.5$ D. $3 \cdot 5 + 2$

Mathematical systems called *fields* are used in telecommunication and cryptology. Every field has at least the elements 0 and 1, and the operations addition + and multiplication ·. The tables below show the operations of a field whose elements are 0, 1, 2 and 3. For instance, $1 + 3 = 2$, because in the addition table, the intersection of the row labelled with the element 1 and the column labelled with the element 3 contains 2.

+	**0**	**1**	**2**	**3**
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

·	**0**	**1**	**2**	**3**
0	0	0	0	0
1	0	1	2	3
2	0	2	3	1
3	0	3	1	2

2. Every element other than 0 has an *inverse element*. The product of an element and its inverse is 1. What is the inverse of the element 2 in the example field? Choose precisely one alternative.

   A. 0 B. 1 C. 2 D. 3

For any field, its every element occurs in the addition table to the right of the vertical line in every row, and below the horizontal line in every column. In the multiplication table, every entry in the row 0 and every entry in the column 0 contains 0, and every element occurs in all remaining rows to the right of the vertical line. For every element of the field, $a + 0 = 0 + a = a$ and $a \cdot 1 = 1 \cdot a = a$ hold.

3. Draw the multiplication table of some 3-element field.
4. Draw the addition table of some 3-element field.
5. For every element a, b and c of any field, $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$ holds. Using only the information provided in this question, justify that the system specified by the tables on the right is not a field.

+	**0**	**1**	**2**	**3**
0	0	1	2	3
1	1	0	3	2
2	2	3	1	0
3	3	2	0	1

·	**0**	**1**	**2**	**3**
0	0	0	0	0
1	0	1	2	3
2	0	2	3	1
3	0	3	1	2

There is a field with precisely n elements if and only if n is either a prime number or some prime number multiplied by itself some number of times. For instance, there is a 4-element field, because $4 = 2 \cdot 2$ and 2 is a prime number. There is no 6-element field, because 6 is neither a prime number, nor the square, cube, and so on, of any prime number.

6. What is the third smallest positive integer such that there is no field with that many elements? In other words, if the sizes bigger than zero that are impossible for fields are ordered in a sequence starting from the smallest, then which positive integer is the third? Choose precisely one alternative. You may get 0 or 0.5 or 1 points from this question.

   A. 6 B. 8 C. 9 D. 10 E. 12 F. 14

WC

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