

Instructions. Reserve a separate page for each problem. Give your solutions in a clear form including intermediate steps. Write a clean copy of the solution if needed. Cross out discarded solutions, in case of two solutions, only the weaker one will be credited.

- The building costs consist only of labour costs and costs of materials. When planning a building, the building costs were estimated as 1 800 000 euros. The final building costs exceeded the estimate by 5.9%. By how many euros did the final costs of materials exceed the estimated costs of materials, when the final labour costs exceeded the estimate by 40 000 euros?
- We consider an attic with a rectangular floor. The longer side of the floor has length s . The figure shows the form of the cross section perpendicular to the longer side. Find the volume of the attic, when $s = 6.90$ m, $a = 1.25$ m, $b = 4.10$ m, $c = 1.10$ m and $d = 0.60$ m.

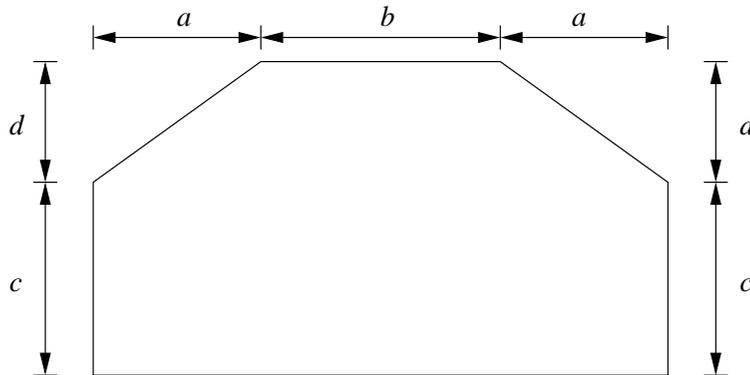


Figure for problem 2, the cross section of the attic.

- The parabola $y = ax^2 + bx + c$ passes through the points $(0, 4)$, $(1, 3)$, and $(2, 0)$. Find the area of the finite region bounded by the parabola and the x -axis.
- An entrance with three doors is constructed as follows. The height of the doors is h . Two doors with width $\frac{1}{2}h$ are placed side by side. The width of the third door is determined by the requirement that the total width of the doors shall be $\frac{1}{2}h + L$, where L is the length of the diagonal of the door in the middle (see figure). Use the height h to express the lengths $|AC|$ and $|BC|$ of the line segments AC and BC .

Show that $|AB| : |AC| = |BC| : |AB|$ (i.e. that the point B divides the width AC according to the golden section).

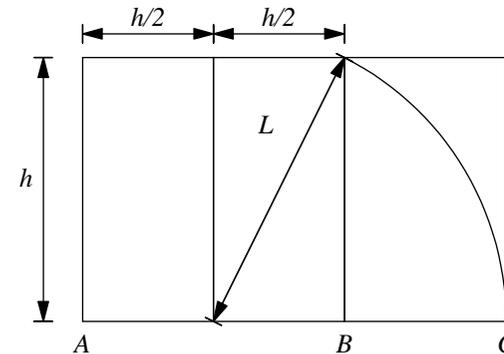


Figure for problem 4

- A vertical pillar has the form of a rectangular parallelepiped. The horizontal cross section of the pillar is a square with side length 2 m. When the pillar is intersected by a plane, the section is a quadrangle $ABCD$, where the corners A , B and C are at the heights 7 m, 8 m and 9 m, respectively. Find the height of point D and the area of the section $ABCD$.
- Five identical drinking glasses in the form of truncated right circular cones are packed into a box with a square bottom. The side length of the bottom square is twice the mouth diameter of the glasses. Four of the glasses are placed so that their mouths rest against the bottom of the box (we assume that the mouth diameter of the glasses is larger than their bottom diameter). The fifth glass is placed in the middle of the other glasses with its bottom downwards. The mouth diameter of the glasses is d , and the height of the glasses is h .
 - How large may the bottom diameter of the glasses be at most, if we want the bottom of the fifth glass to touch the bottom of the box?
 - How high above the bottom of the box is the bottom of the fifth glass, if the bottom diameter of the glasses is $\frac{1}{2}d$?