**Entrance examination in mathematics May 23, 2005**

**Instructions.** Reserve a *separate page* for each problem. Give your solutions in a clear form *including intermediate steps*. Write a clean copy of the solution if needed. *Cross out discarded solutions* (in case of several solutions of the same problem, only the weakest one will be credited).

1. Albert's student allowance is 22 % less than his necessary living costs. The government plans to raise the allowance for the year 2006 according to the progress of studies so that the raise percentage is half of the credits obtained in 2005. Assuming that the plan is followed, how many credits should Albert obtain during 2005 so that his allowance would be at least equal to his living costs ?

2. Determine the area of the triangle in the $xy$-plane bounded by the coordinate axes and the tangent line to the curve $y = 3/x$ $(x > 0)$ at the point $(x_0, y_0)$.

3. A cotton reel has the shape of a solid of revolution, with a hole in the middle, and it is produced following the sketch below (in the diagram the lengths are given in millimeters). Calculate the weight of the cotton reel when the dry density of the wood it is made of is 0.495 g/cm$^3$. Give the answer in grams with two decimal places.

4. A ball as large as possible is placed inside a cube. The cube and the ball are intersected with a plane which passes through one edge of the cube and divides one face of the cube in two rectangles of equal areas. Draw a diagram where both the intersection of the plane and the cube as well as the intersection of the plane and the ball appear together in the actual shape. Calculate also the necessary measures of the diagram when the length of the edge of the cube is 6 cm.

5. In a certain country the number $m = m(t)$ of hiv-infected people is estimated to have the rate of change
$$m'(t) = 24(t - \frac{5}{200}t^2)$$
for $0 \le t \le 20$ (time $t$ is given in years). When the follow-up study begins $(t = 0)$, the number of infected is 25. How many people will be infected after 10 years according to this model? And how many more people will get the infection during the next 10 years after that?

6. The walls of an observatory are built in the shape of a right circular cylinder with radius $r$. The observatory also has a floor and a hemispherical roof with radius $r$. How should one choose the radius $r$ in order to minimize the total area of walls, floor and roof, if the volume of the tower is 1440 m$^3$?
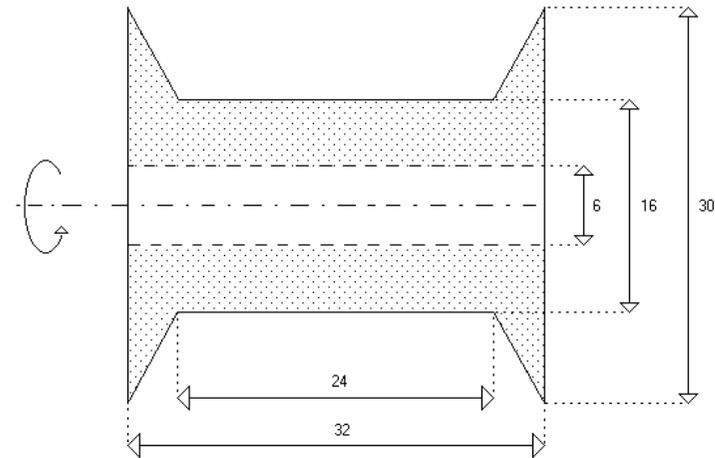
Appendix: Table of Formulas



Figure (problem 3.)