

DIA-valinta 2009

ArkMat en

sarja **A**

18.5.2009

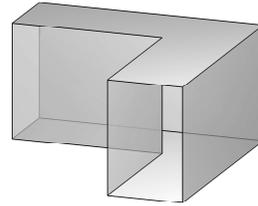
nippukoko 10 (1/0)

Instructions. Reserve a separate page for each problem. Indicate if the answer continues on a separate sheet. Give your solutions in a clear form *including intermediate steps*. Rewrite a clean copy of the solution if needed. *Cross out discarded solutions and any discarded parts of the solutions*. In the case of several solutions for the same problem, only the weakest one will be credited. Note that subsections of a question are not necessarily equally weighted.

Allowed instruments: Writing instruments, non-programmable calculators, non-electronic general-language dictionaries to/from English.

Attachment: Table of formulae.

- A1 A corridor makes a 90 degree turn as illustrated in the attached figure. The corridor's cross section is a rectangle. The width of the corridor is $d = 2,0$ m and the height $h = 3,0$ m. How long a bar can be transported through the corridor without bending it?



The thickness of the bar can be neglected. Give the answer rounded to integral centimeters.

- A2 Consider a piece of land having the form of a right-angled triangle, where the length of the two shorter edges are $a = 37$ m, $b = 41$ m. What is the land area free for constructing when one is required to leave a 3 m wide strip unexploited along the entire length of the longest edge.

Give the answer rounded to the accuracy of 0.1 m^2 .

- A3 Architects M. Uoto, F. Örm, and S. Hapé have designed a monument consisting of a gilded steel cube to be placed in the entré of Aalto-university. The gilding is thin.

During the execution of the project, the planned location for the monument is altered, whereby the volume of the steel cube is increased by 19% compared to the original size. In the final billing, the cost of the materials is also found to be 19% larger than that in the original budget.

- How much did the amount of gold needed for the gilding grow during the execution of the project?
- The unit price of the steel did not change during the execution of the project. Thus, how did the unit price of gold change?

Give the answers in percent rounded to the accuracy of 0.1 percentage points.

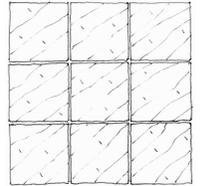
- A4 On the bottom of a hemisphere formed champagne bowl lies a cooled steel ball. The bowl is filled. What fraction of the champagne is poured out of the bowl, as the bowl is tilted 45° ? The inner diameter of the bowl is D and the diameter of the ball $d = \frac{1}{2}D$.

(The volume of a segment of a sphere of radius R is given by $V = \pi h^2(R - \frac{h}{3})$.)

Give the exact answer and its approximation rounded to the accuracy of three decimal places.

- A5 To commemorate the Finnish War (Swedish-Russian war 1809) one arranges a mosaic placed on a wall. The mosaic is made of nine square ceramic tiles, which are placed at even intervals in the mosaic. Each tile is chosen randomly from an infinite pile of blue and yellow tiles. Of the tiles, 60% are blue.

- What is the probability that the mosaic is bi-coloured?
- What is the probability that the color pattern of the mosaic is not altered, if its tiles are rearranged at positions rotated 90 degrees clockwise compared to the original ones?

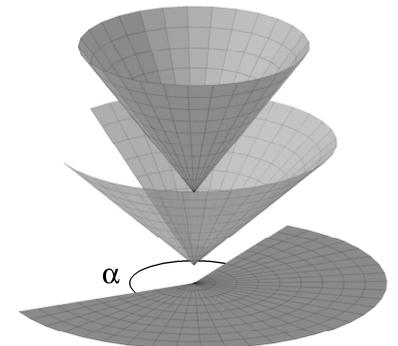


Give the probabilities rounded to the accuracy of three decimal places.

- A6 A piece of cardboard has the form of disc of radius r . From the piece one cuts away a sector with a central angle α . The remaining piece of cardboard is bent to form a circular cone such that the cut edges are placed edge-to-edge.

- Find the volume of the circular cone, when $\alpha = 90^\circ$ and $r = 10$ cm?
- How should one choose α in order to maximize the volume of the circular cone?

Give the answers rounded to the accuracy of 0,1 cubic centimeters and 0,1 degrees respectively.

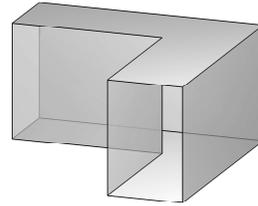


Instructions. Reserve a separate page for each problem. Indicate if the answer continues on a separate sheet. Give your solutions in a clear form *including intermediate steps*. Rewrite a clean copy of the solution if needed. *Cross out discarded solutions and any discarded parts of the solutions*. In the case of several solutions for the same problem, only the weakest one will be credited. Note that subsections of a question are not necessarily equally weighted.

Allowed instruments: Writing instruments, non-programmable calculators, non-electronic general-language dictionaries to/from English.

Attachment: Table of formulae.

- A1 A corridor makes a 90 degree turn as illustrated in the attached figure. The corridor's cross section is a rectangle. The width of the corridor is $d = 2,0$ m and the height $h = 3,0$ m. How long a bar can be transported through the corridor without bending it?



The thickness of the bar can be neglected. Give the answer rounded to integral centimeters.

- A2 Consider a piece of land having the form of a right-angled triangle, where the length of the two shorter edges are $a = 37$ m, $b = 41$ m. What is the land area free for constructing when one is required to leave a 3 m wide strip unexploited along the entire length of the longest edge.

Give the answer rounded to the accuracy of 0.1 m^2 .

- A3 Architects M. Uoto, F. Örm, and S. Hapé have designed a monument consisting of a gilded steel cube to be placed in the entré of Aalto-university. The gilding is thin.

During the execution of the project, the planned location for the monument is altered, whereby the volume of the steel cube is increased by 19% compared to the original size. In the final billing, the cost of the materials is also found to be 19% larger than that in the original budget.

- How much did the amount of gold needed for the gilding grow during the execution of the project?
- The unit price of the steel did not change during the execution of the project. Thus, how did the unit price of gold change?

Give the answers in percent rounded to the accuracy of 0.1 percentage points.

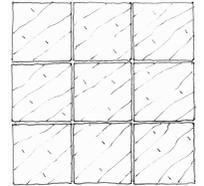
- A4 On the bottom of a hemisphere formed champagne bowl lies a cooled steel ball. The bowl is filled. What fraction of the champagne is poured out of the bowl, as the bowl is tilted 45° ? The inner diameter of the bowl is D and the diameter of the ball $d = \frac{1}{2}D$.

(The volume of a segment of a sphere of radius R is given by $V = \pi h^2(R - \frac{h}{3})$.)

Give the exact answer and its approximation rounded to the accuracy of three decimal places.

- A5 To commemorate the Finnish War (Swedish-Russian war 1809) one arranges a mosaic placed on a wall. The mosaic is made of nine square ceramic tiles, which are placed at even intervals in the mosaic. Each tile is chosen randomly from an infinite pile of blue and yellow tiles. Of the tiles, 60% are blue.

- What is the probability that the mosaic is bi-coloured?
- What is the probability that the color pattern of the mosaic is not altered, if its tiles are rearranged at positions rotated 90 degrees clockwise compared to the original ones?

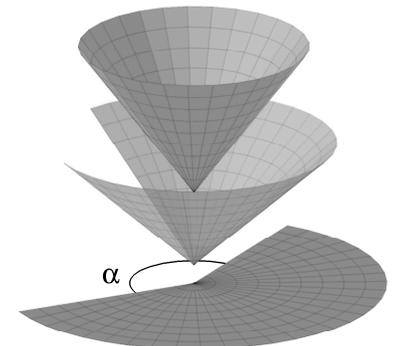


Give the probabilities rounded to the accuracy of three decimal places.

- A6 A piece of cardboard has the form of disc of radius r . From the piece one cuts away a sector with a central angle α . The remaining piece of cardboard is bent to form a circular cone such that the cut edges are placed edge-to-edge.

- Find the volume of the circular cone, when $\alpha = 90^\circ$ and $r = 10$ cm?
- How should one choose α in order to maximize the volume of the circular cone?

Give the answers rounded to the accuracy of 0,1 cubic centimeters and 0,1 degrees respectively.

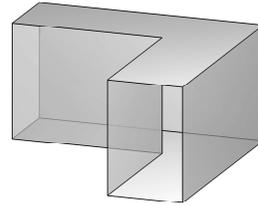


Instructions. Reserve a separate page for each problem. Indicate if the answer continues on a separate sheet. Give your solutions in a clear form including intermediate steps. Rewrite a clean copy of the solution if needed. Cross out discarded solutions and any discarded parts of the solutions. In the case of several solutions for the same problem, only the weakest one will be credited. Note that subsections of a question are not necessarily equally weighted.

Allowed instruments: Writing instruments, non-programmable calculators, non-electronic general-language dictionaries to/from English.

Attachment: Table of formulae.

- A1 A corridor makes a 90 degree turn as illustrated in the attached figure. The corridor's cross section is a rectangle. The width of the corridor is $d = 2,0$ m and the height $h = 3,0$ m. How long a bar can be transported through the corridor without bending it?



The thickness of the bar can be neglected. Give the answer rounded to integral centimeters.

- A2 Consider a piece of land having the form of a right-angled triangle, where the length of the two shorter edges are $a = 37$ m, $b = 41$ m. What is the land area free for constructing when one is required to leave a 3 m wide strip unexploited along the entire length of the longest edge.

Give the answer rounded to the accuracy of 0.1 m^2 .

- A3 Architects M. Uoto, F. Örm, and S. Hapé have designed a monument consisting of a gilded steel cube to be placed in the entré of Aalto-university. The gilding is thin.

During the execution of the project, the planned location for the monument is altered, whereby the volume of the steel cube is increased by 19% compared to the original size. In the final billing, the cost of the materials is also found to be 19% larger than that in the original budget.

- How much did the amount of gold needed for the gilding grow during the execution of the project?
- The unit price of the steel did not change during the execution of the project. Thus, how did the unit price of gold change?

Give the answers in percent rounded to the accuracy of 0.1 percentage points.

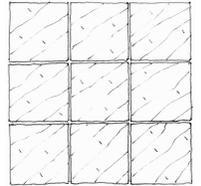
- A4 On the bottom of a hemisphere formed champagne bowl lies a cooled steel ball. The bowl is filled. What fraction of the champagne is poured out of the bowl, as the bowl is tilted 45° ? The inner diameter of the bowl is D and the diameter of the ball $d = \frac{1}{2}D$.

(The volume of a segment of a sphere of radius R is given by $V = \pi h^2(R - \frac{h}{3})$.)

Give the exact answer and its approximation rounded to the accuracy of three decimal places.

- A5 To commemorate the Finnish War (Swedish-Russian war 1809) one arranges a mosaic placed on a wall. The mosaic is made of nine square ceramic tiles, which are placed at even intervals in the mosaic. Each tile is chosen randomly from an infinite pile of blue and yellow tiles. Of the tiles, 60% are blue.

- What is the probability that the mosaic is bi-coloured?
- What is the probability that the color pattern of the mosaic is not altered, if its tiles are rearranged at positions rotated 90 degrees clockwise compared to the original ones?

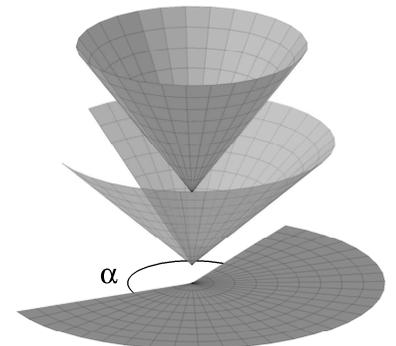


Give the probabilities rounded to the accuracy of three decimal places.

- A6 A piece of cardboard has the form of disc of radius r . From the piece one cuts away a sector with a central angle α . The remaining piece of cardboard is bent to form a circular cone such that the cut edges are placed edge-to-edge.

- Find the volume of the circular cone, when $\alpha = 90^\circ$ and $r = 10$ cm?
- How should one choose α in order to maximize the volume of the circular cone?

Give the answers rounded to the accuracy of 0,1 cubic centimeters and 0,1 degrees respectively.

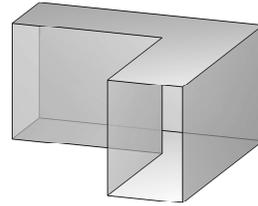


Instructions. Reserve a separate page for each problem. Indicate if the answer continues on a separate sheet. Give your solutions in a clear form *including intermediate steps*. Rewrite a clean copy of the solution if needed. *Cross out discarded solutions and any discarded parts of the solutions*. In the case of several solutions for the same problem, only the weakest one will be credited. Note that subsections of a question are not necessarily equally weighted.

Allowed instruments: Writing instruments, non-programmable calculators, non-electronic general-language dictionaries to/from English.

Attachment: Table of formulae.

- A1 A corridor makes a 90 degree turn as illustrated in the attached figure. The corridor's cross section is a rectangle. The width of the corridor is $d = 2,0$ m and the height $h = 3,0$ m. How long a bar can be transported through the corridor without bending it?



The thickness of the bar can be neglected. Give the answer rounded to integral centimeters.

- A2 Consider a piece of land having the form of a right-angled triangle, where the length of the two shorter edges are $a = 37$ m, $b = 41$ m. What is the land area free for constructing when one is required to leave a 3 m wide strip unexploited along the entire length of the longest edge.

Give the answer rounded to the accuracy of 0.1 m^2 .

- A3 Architects M. Uoto, F. Örm, and S. Hapé have designed a monument consisting of a gilded steel cube to be placed in the entré of Aalto-university. The gilding is thin.

During the execution of the project, the planned location for the monument is altered, whereby the volume of the steel cube is increased by 19% compared to the original size. In the final billing, the cost of the materials is also found to be 19% larger than that in the original budget.

- How much did the amount of gold needed for the gilding grow during the execution of the project?
- The unit price of the steel did not change during the execution of the project. Thus, how did the unit price of gold change?

Give the answers in percent rounded to the accuracy of 0.1 percentage points.

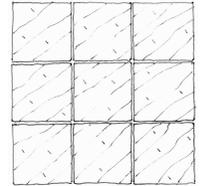
- A4 On the bottom of a hemisphere formed champagne bowl lies a cooled steel ball. The bowl is filled. What fraction of the champagne is poured out of the bowl, as the bowl is tilted 45° ? The inner diameter of the bowl is D and the diameter of the ball $d = \frac{1}{2}D$.

(The volume of a segment of a sphere of radius R is given by $V = \pi h^2(R - \frac{h}{3})$.)

Give the exact answer and its approximation rounded to the accuracy of three decimal places.

- A5 To commemorate the Finnish War (Swedish-Russian war 1809) one arranges a mosaic placed on a wall. The mosaic is made of nine square ceramic tiles, which are placed at even intervals in the mosaic. Each tile is chosen randomly from an infinite pile of blue and yellow tiles. Of the tiles, 60% are blue.

- What is the probability that the mosaic is bi-coloured?
- What is the probability that the color pattern of the mosaic is not altered, if its tiles are rearranged at positions rotated 90 degrees clockwise compared to the original ones?

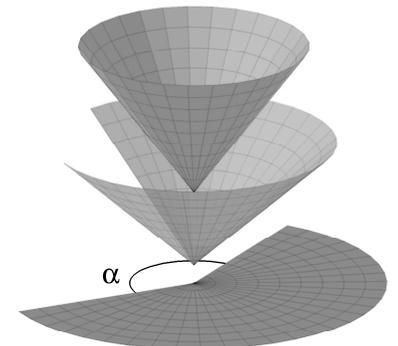


Give the probabilities rounded to the accuracy of three decimal places.

- A6 A piece of cardboard has the form of disc of radius r . From the piece one cuts away a sector with a central angle α . The remaining piece of cardboard is bent to form a circular cone such that the cut edges are placed edge-to-edge.

- Find the volume of the circular cone, when $\alpha = 90^\circ$ and $r = 10$ cm?
- How should one choose α in order to maximize the volume of the circular cone?

Give the answers rounded to the accuracy of 0,1 cubic centimeters and 0,1 degrees respectively.

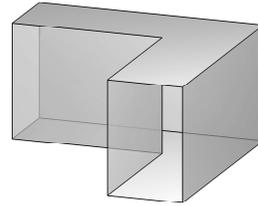


Instructions. Reserve a separate page for each problem. Indicate if the answer continues on a separate sheet. Give your solutions in a clear form *including intermediate steps*. Rewrite a clean copy of the solution if needed. *Cross out discarded solutions and any discarded parts of the solutions*. In the case of several solutions for the same problem, only the weakest one will be credited. Note that subsections of a question are not necessarily equally weighted.

Allowed instruments: Writing instruments, non-programmable calculators, non-electronic general-language dictionaries to/from English.

Attachment: Table of formulae.

- A1 A corridor makes a 90 degree turn as illustrated in the attached figure. The corridor's cross section is a rectangle. The width of the corridor is $d = 2,0$ m and the height $h = 3,0$ m. How long a bar can be transported through the corridor without bending it?



The thickness of the bar can be neglected. Give the answer rounded to integral centimeters.

- A2 Consider a piece of land having the form of a right-angled triangle, where the length of the two shorter edges are $a = 37$ m, $b = 41$ m. What is the land area free for constructing when one is required to leave a 3 m wide strip unexploited along the entire length of the longest edge.

Give the answer rounded to the accuracy of 0.1 m^2 .

- A3 Architects M. Uoto, F. Örm, and S. Hapé have designed a monument consisting of a gilded steel cube to be placed in the entré of Aalto-university. The gilding is thin.

During the execution of the project, the planned location for the monument is altered, whereby the volume of the steel cube is increased by 19% compared to the original size. In the final billing, the cost of the materials is also found to be 19% larger than that in the original budget.

- How much did the amount of gold needed for the gilding grow during the execution of the project?
- The unit price of the steel did not change during the execution of the project. Thus, how did the unit price of gold change?

Give the answers in percent rounded to the accuracy of 0.1 percentage points.

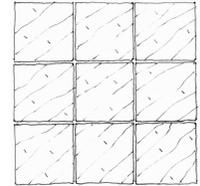
- A4 On the bottom of a hemisphere formed champagne bowl lies a cooled steel ball. The bowl is filled. What fraction of the champagne is poured out of the bowl, as the bowl is tilted 45° ? The inner diameter of the bowl is D and the diameter of the ball $d = \frac{1}{2}D$.

(The volume of a segment of a sphere of radius R is given by $V = \pi h^2(R - \frac{h}{3})$.)

Give the exact answer and its approximation rounded to the accuracy of three decimal places.

- A5 To commemorate the Finnish War (Swedish-Russian war 1809) one arranges a mosaic placed on a wall. The mosaic is made of nine square ceramic tiles, which are placed at even intervals in the mosaic. Each tile is chosen randomly from an infinite pile of blue and yellow tiles. Of the tiles, 60% are blue.

- What is the probability that the mosaic is bi-coloured?
- What is the probability that the color pattern of the mosaic is not altered, if its tiles are rearranged at positions rotated 90 degrees clockwise compared to the original ones?

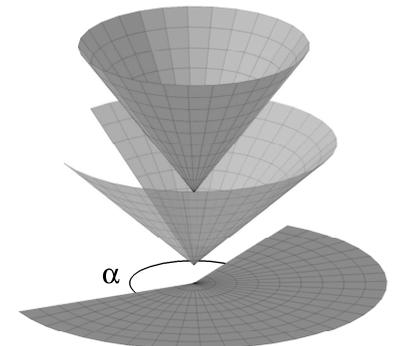


Give the probabilities rounded to the accuracy of three decimal places.

- A6 A piece of cardboard has the form of disc of radius r . From the piece one cuts away a sector with a central angle α . The remaining piece of cardboard is bent to form a circular cone such that the cut edges are placed edge-to-edge.

- Find the volume of the circular cone, when $\alpha = 90^\circ$ and $r = 10$ cm?
- How should one choose α in order to maximize the volume of the circular cone?

Give the answers rounded to the accuracy of 0,1 cubic centimeters and 0,1 degrees respectively.

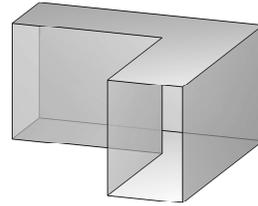


Instructions. Reserve a separate page for each problem. Indicate if the answer continues on a separate sheet. Give your solutions in a clear form *including intermediate steps*. Rewrite a clean copy of the solution if needed. *Cross out discarded solutions and any discarded parts of the solutions*. In the case of several solutions for the same problem, only the weakest one will be credited. Note that subsections of a question are not necessarily equally weighted.

Allowed instruments: Writing instruments, non-programmable calculators, non-electronic general-language dictionaries to/from English.

Attachment: Table of formulae.

- A1 A corridor makes a 90 degree turn as illustrated in the attached figure. The corridor's cross section is a rectangle. The width of the corridor is $d = 2,0$ m and the height $h = 3,0$ m. How long a bar can be transported through the corridor without bending it?



The thickness of the bar can be neglected. Give the answer rounded to integral centimeters.

- A2 Consider a piece of land having the form of a right-angled triangle, where the length of the two shorter edges are $a = 37$ m, $b = 41$ m. What is the land area free for constructing when one is required to leave a 3 m wide strip unexploited along the entire length of the longest edge.

Give the answer rounded to the accuracy of 0.1 m^2 .

- A3 Architects M. Uoto, F. Örm, and S. Hapé have designed a monument consisting of a gilded steel cube to be placed in the entré of Aalto-university. The gilding is thin.

During the execution of the project, the planned location for the monument is altered, whereby the volume of the steel cube is increased by 19% compared to the original size. In the final billing, the cost of the materials is also found to be 19% larger than that in the original budget.

- How much did the amount of gold needed for the gilding grow during the execution of the project?
- The unit price of the steel did not change during the execution of the project. Thus, how did the unit price of gold change?

Give the answers in percent rounded to the accuracy of 0.1 percentage points.

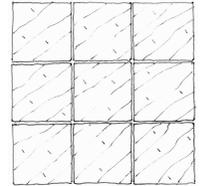
- A4 On the bottom of a hemisphere formed champagne bowl lies a cooled steel ball. The bowl is filled. What fraction of the champagne is poured out of the bowl, as the bowl is tilted 45° ? The inner diameter of the bowl is D and the diameter of the ball $d = \frac{1}{2}D$.

(The volume of a segment of a sphere of radius R is given by $V = \pi h^2(R - \frac{h}{3})$.)

Give the exact answer and its approximation rounded to the accuracy of three decimal places.

- A5 To commemorate the Finnish War (Swedish-Russian war 1809) one arranges a mosaic placed on a wall. The mosaic is made of nine square ceramic tiles, which are placed at even intervals in the mosaic. Each tile is chosen randomly from an infinite pile of blue and yellow tiles. Of the tiles, 60% are blue.

- What is the probability that the mosaic is bi-coloured?
- What is the probability that the color pattern of the mosaic is not altered, if its tiles are rearranged at positions rotated 90 degrees clockwise compared to the original ones?

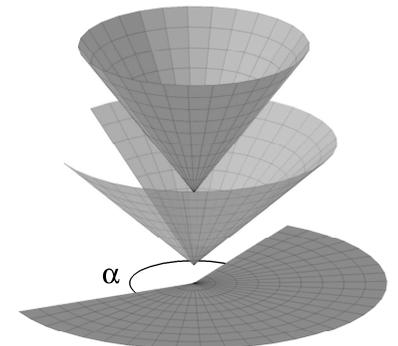


Give the probabilities rounded to the accuracy of three decimal places.

- A6 A piece of cardboard has the form of disc of radius r . From the piece one cuts away a sector with a central angle α . The remaining piece of cardboard is bent to form a circular cone such that the cut edges are placed edge-to-edge.

- Find the volume of the circular cone, when $\alpha = 90^\circ$ and $r = 10$ cm?
- How should one choose α in order to maximize the volume of the circular cone?

Give the answers rounded to the accuracy of 0,1 cubic centimeters and 0,1 degrees respectively.

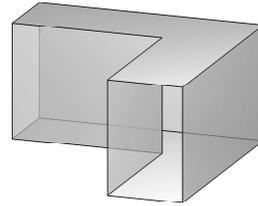


Instructions. Reserve a separate page for each problem. Indicate if the answer continues on a separate sheet. Give your solutions in a clear form including intermediate steps. Rewrite a clean copy of the solution if needed. Cross out discarded solutions and any discarded parts of the solutions. In the case of several solutions for the same problem, only the weakest one will be credited. Note that subsections of a question are not necessarily equally weighted.

Allowed instruments: Writing instruments, non-programmable calculators, non-electronic general-language dictionaries to/from English.

Attachment: Table of formulae.

- A1 A corridor makes a 90 degree turn as illustrated in the attached figure. The corridor's cross section is a rectangle. The width of the corridor is $d = 2,0$ m and the height $h = 3,0$ m. How long a bar can be transported through the corridor without bending it?



The thickness of the bar can be neglected. Give the answer rounded to integral centimeters.

- A2 Consider a piece of land having the form of a right-angled triangle, where the length of the two shorter edges are $a = 37$ m, $b = 41$ m. What is the land area free for constructing when one is required to leave a 3 m wide strip unexploited along the entire length of the longest edge.

Give the answer rounded to the accuracy of 0.1 m^2 .

- A3 Architects M. Uoto, F. Örm, and S. Hapé have designed a monument consisting of a gilded steel cube to be placed in the entré of Aalto-university. The gilding is thin.

During the execution of the project, the planned location for the monument is altered, whereby the volume of the steel cube is increased by 19% compared to the original size. In the final billing, the cost of the materials is also found to be 19% larger than that in the original budget.

- How much did the amount of gold needed for the gilding grow during the execution of the project?
- The unit price of the steel did not change during the execution of the project. Thus, how did the unit price of gold change?

Give the answers in percent rounded to the accuracy of 0.1 percentage points.

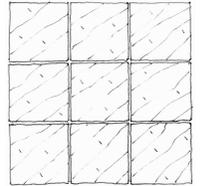
- A4 On the bottom of a hemisphere formed champagne bowl lies a cooled steel ball. The bowl is filled. What fraction of the champagne is poured out of the bowl, as the bowl is tilted 45° ? The inner diameter of the bowl is D and the diameter of the ball $d = \frac{1}{2}D$.

(The volume of a segment of a sphere of radius R is given by $V = \pi h^2(R - \frac{h}{3})$.)

Give the exact answer and its approximation rounded to the accuracy of three decimal places.

- A5 To commemorate the Finnish War (Swedish-Russian war 1809) one arranges a mosaic placed on a wall. The mosaic is made of nine square ceramic tiles, which are placed at even intervals in the mosaic. Each tile is chosen randomly from an infinite pile of blue and yellow tiles. Of the tiles, 60% are blue.

- What is the probability that the mosaic is bi-coloured?
- What is the probability that the color pattern of the mosaic is not altered, if its tiles are rearranged at positions rotated 90 degrees clockwise compared to the original ones?

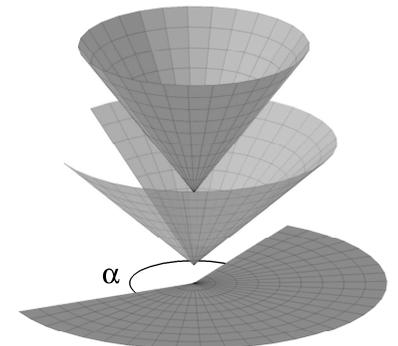


Give the probabilities rounded to the accuracy of three decimal places.

- A6 A piece of cardboard has the form of disc of radius r . From the piece one cuts away a sector with a central angle α . The remaining piece of cardboard is bent to form a circular cone such that the cut edges are placed edge-to-edge.

- Find the volume of the circular cone, when $\alpha = 90^\circ$ and $r = 10$ cm?
- How should one choose α in order to maximize the volume of the circular cone?

Give the answers rounded to the accuracy of 0,1 cubic centimeters and 0,1 degrees respectively.

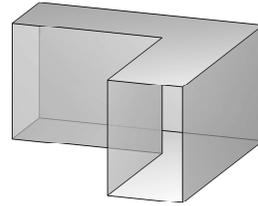


Instructions. Reserve a separate page for each problem. Indicate if the answer continues on a separate sheet. Give your solutions in a clear form *including intermediate steps*. Rewrite a clean copy of the solution if needed. *Cross out discarded solutions and any discarded parts of the solutions*. In the case of several solutions for the same problem, only the weakest one will be credited. Note that subsections of a question are not necessarily equally weighted.

Allowed instruments: Writing instruments, non-programmable calculators, non-electronic general-language dictionaries to/from English.

Attachment: Table of formulae.

- A1 A corridor makes a 90 degree turn as illustrated in the attached figure. The corridor's cross section is a rectangle. The width of the corridor is $d = 2,0$ m and the height $h = 3,0$ m. How long a bar can be transported through the corridor without bending it?



The thickness of the bar can be neglected. Give the answer rounded to integral centimeters.

- A2 Consider a piece of land having the form of a right-angled triangle, where the length of the two shorter edges are $a = 37$ m, $b = 41$ m. What is the land area free for constructing when one is required to leave a 3 m wide strip unexploited along the entire length of the longest edge.

Give the answer rounded to the accuracy of 0.1 m^2 .

- A3 Architects M. Uoto, F. Örm, and S. Hapé have designed a monument consisting of a gilded steel cube to be placed in the entré of Aalto-university. The gilding is thin.

During the execution of the project, the planned location for the monument is altered, whereby the volume of the steel cube is increased by 19% compared to the original size. In the final billing, the cost of the materials is also found to be 19% larger than that in the original budget.

- How much did the amount of gold needed for the gilding grow during the execution of the project?
- The unit price of the steel did not change during the execution of the project. Thus, how did the unit price of gold change?

Give the answers in percent rounded to the accuracy of 0.1 percentage points.

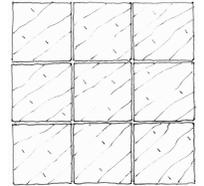
- A4 On the bottom of a hemisphere formed champagne bowl lies a cooled steel ball. The bowl is filled. What fraction of the champagne is poured out of the bowl, as the bowl is tilted 45° ? The inner diameter of the bowl is D and the diameter of the ball $d = \frac{1}{2}D$.

(The volume of a segment of a sphere of radius R is given by $V = \pi h^2(R - \frac{h}{3})$.)

Give the exact answer and its approximation rounded to the accuracy of three decimal places.

- A5 To commemorate the Finnish War (Swedish-Russian war 1809) one arranges a mosaic placed on a wall. The mosaic is made of nine square ceramic tiles, which are placed at even intervals in the mosaic. Each tile is chosen randomly from an infinite pile of blue and yellow tiles. Of the tiles, 60% are blue.

- What is the probability that the mosaic is bi-coloured?
- What is the probability that the color pattern of the mosaic is not altered, if its tiles are rearranged at positions rotated 90 degrees clockwise compared to the original ones?

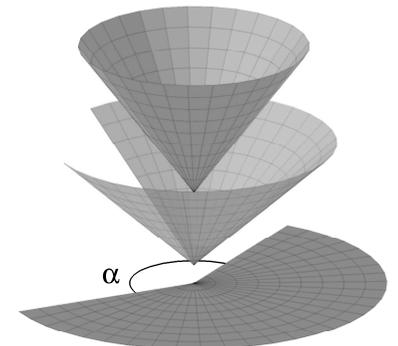


Give the probabilities rounded to the accuracy of three decimal places.

- A6 A piece of cardboard has the form of disc of radius r . From the piece one cuts away a sector with a central angle α . The remaining piece of cardboard is bent to form a circular cone such that the cut edges are placed edge-to-edge.

- Find the volume of the circular cone, when $\alpha = 90^\circ$ and $r = 10$ cm?
- How should one choose α in order to maximize the volume of the circular cone?

Give the answers rounded to the accuracy of 0,1 cubic centimeters and 0,1 degrees respectively.

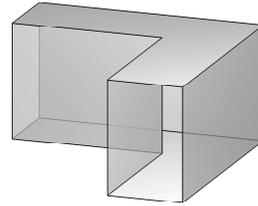


Instructions. Reserve a separate page for each problem. Indicate if the answer continues on a separate sheet. Give your solutions in a clear form *including intermediate steps*. Rewrite a clean copy of the solution if needed. *Cross out discarded solutions and any discarded parts of the solutions*. In the case of several solutions for the same problem, only the weakest one will be credited. Note that subsections of a question are not necessarily equally weighted.

Allowed instruments: Writing instruments, non-programmable calculators, non-electronic general-language dictionaries to/from English.

Attachment: Table of formulae.

- A1 A corridor makes a 90 degree turn as illustrated in the attached figure. The corridor's cross section is a rectangle. The width of the corridor is $d = 2,0$ m and the height $h = 3,0$ m. How long a bar can be transported through the corridor without bending it?



The thickness of the bar can be neglected. Give the answer rounded to integral centimeters.

- A2 Consider a piece of land having the form of a right-angled triangle, where the length of the two shorter edges are $a = 37$ m, $b = 41$ m. What is the land area free for constructing when one is required to leave a 3 m wide strip unexploited along the entire length of the longest edge.

Give the answer rounded to the accuracy of 0.1 m^2 .

- A3 Architects M. Uoto, F. Örm, and S. Hapé have designed a monument consisting of a gilded steel cube to be placed in the entré of Aalto-university. The gilding is thin.

During the execution of the project, the planned location for the monument is altered, whereby the volume of the steel cube is increased by 19% compared to the original size. In the final billing, the cost of the materials is also found to be 19% larger than that in the original budget.

- How much did the amount of gold needed for the gilding grow during the execution of the project?
- The unit price of the steel did not change during the execution of the project. Thus, how did the unit price of gold change?

Give the answers in percent rounded to the accuracy of 0.1 percentage points.

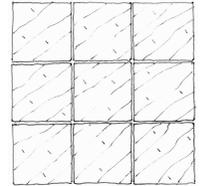
- A4 On the bottom of a hemisphere formed champagne bowl lies a cooled steel ball. The bowl is filled. What fraction of the champagne is poured out of the bowl, as the bowl is tilted 45° ? The inner diameter of the bowl is D and the diameter of the ball $d = \frac{1}{2}D$.

(The volume of a segment of a sphere of radius R is given by $V = \pi h^2(R - \frac{h}{3})$.)

Give the exact answer and its approximation rounded to the accuracy of three decimal places.

- A5 To commemorate the Finnish War (Swedish-Russian war 1809) one arranges a mosaic placed on a wall. The mosaic is made of nine square ceramic tiles, which are placed at even intervals in the mosaic. Each tile is chosen randomly from an infinite pile of blue and yellow tiles. Of the tiles, 60% are blue.

- What is the probability that the mosaic is bi-coloured?
- What is the probability that the color pattern of the mosaic is not altered, if its tiles are rearranged at positions rotated 90 degrees clockwise compared to the original ones?

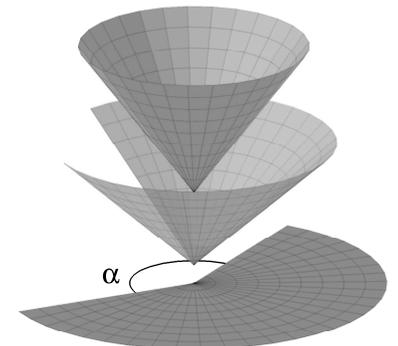


Give the probabilities rounded to the accuracy of three decimal places.

- A6 A piece of cardboard has the form of disc of radius r . From the piece one cuts away a sector with a central angle α . The remaining piece of cardboard is bent to form a circular cone such that the cut edges are placed edge-to-edge.

- Find the volume of the circular cone, when $\alpha = 90^\circ$ and $r = 10$ cm?
- How should one choose α in order to maximize the volume of the circular cone?

Give the answers rounded to the accuracy of 0,1 cubic centimeters and 0,1 degrees respectively.

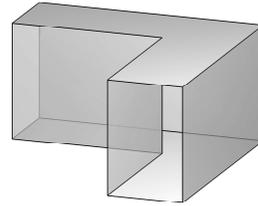


Instructions. Reserve a separate page for each problem. Indicate if the answer continues on a separate sheet. Give your solutions in a clear form including intermediate steps. Rewrite a clean copy of the solution if needed. Cross out discarded solutions and any discarded parts of the solutions. In the case of several solutions for the same problem, only the weakest one will be credited. Note that subsections of a question are not necessarily equally weighted.

Allowed instruments: Writing instruments, non-programmable calculators, non-electronic general-language dictionaries to/from English.

Attachment: Table of formulae.

- A1 A corridor makes a 90 degree turn as illustrated in the attached figure. The corridor's cross section is a rectangle. The width of the corridor is $d = 2,0$ m and the height $h = 3,0$ m. How long a bar can be transported through the corridor without bending it?



The thickness of the bar can be neglected. Give the answer rounded to integral centimeters.

- A2 Consider a piece of land having the form of a right-angled triangle, where the length of the two shorter edges are $a = 37$ m, $b = 41$ m. What is the land area free for constructing when one is required to leave a 3 m wide strip unexploited along the entire length of the longest edge.

Give the answer rounded to the accuracy of 0.1 m^2 .

- A3 Architects M. Uoto, F. Örm, and S. Hapé have designed a monument consisting of a gilded steel cube to be placed in the entré of Aalto-university. The gilding is thin.

During the execution of the project, the planned location for the monument is altered, whereby the volume of the steel cube is increased by 19% compared to the original size. In the final billing, the cost of the materials is also found to be 19% larger than that in the original budget.

- How much did the amount of gold needed for the gilding grow during the execution of the project?
- The unit price of the steel did not change during the execution of the project. Thus, how did the unit price of gold change?

Give the answers in percent rounded to the accuracy of 0.1 percentage points.

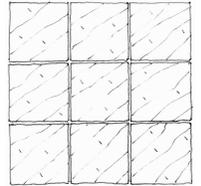
- A4 On the bottom of a hemisphere formed champagne bowl lies a cooled steel ball. The bowl is filled. What fraction of the champagne is poured out of the bowl, as the bowl is tilted 45° ? The inner diameter of the bowl is D and the diameter of the ball $d = \frac{1}{2}D$.

(The volume of a segment of a sphere of radius R is given by $V = \pi h^2(R - \frac{h}{3})$.)

Give the exact answer and its approximation rounded to the accuracy of three decimal places.

- A5 To commemorate the Finnish War (Swedish-Russian war 1809) one arranges a mosaic placed on a wall. The mosaic is made of nine square ceramic tiles, which are placed at even intervals in the mosaic. Each tile is chosen randomly from an infinite pile of blue and yellow tiles. Of the tiles, 60% are blue.

- What is the probability that the mosaic is bi-coloured?
- What is the probability that the color pattern of the mosaic is not altered, if its tiles are rearranged at positions rotated 90 degrees clockwise compared to the original ones?



Give the probabilities rounded to the accuracy of three decimal places.

- A6 A piece of cardboard has the form of disc of radius r . From the piece one cuts away a sector with a central angle α . The remaining piece of cardboard is bent to form a circular cone such that the cut edges are placed edge-to-edge.

- Find the volume of the circular cone, when $\alpha = 90^\circ$ and $r = 10$ cm?
- How should one choose α in order to maximize the volume of the circular cone?

Give the answers rounded to the accuracy of 0,1 cubic centimeters and 0,1 degrees respectively.

