

Exam on architect mathematics 5.6.2024

Instruction: Write your name and identity number clearly on the upper edge of each answer sheet. Start answering on the size sheet (folded A3) and if needed, continue on separate semi-sheets (A4). Mark it clearly if the answer continues to the next sheet. Justify your answers. Place the separate semi-sheets in between the size sheet when returning the exams answers. Tools: Writing instruments and a pocket calculator or a scientific calculator.

Question 1. The price of electricity rose by 20%. By how many percent the consumption can rise at most, if the electricity costs must not rise more than 30%? (6 p.)

Model response: Let M be the electricity cost before the price increase, and let p be the percentage of consumption growth. Then the new costs are $M \cdot 1,2 \cdot (1 + \frac{p}{100})$. This can be at most $M \cdot 1,3$, whence

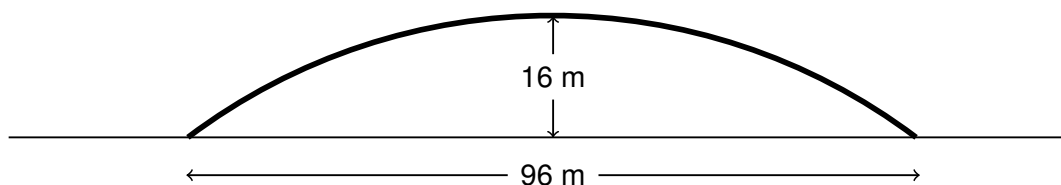
$$M \cdot 1,2 \cdot (1 + \frac{p}{100}) = M \cdot 1,3 \Leftrightarrow 1 + \frac{p}{100} = \frac{1,3}{1,2} \Leftrightarrow p = 8,333 \dots$$

Question 2. The annual population increase in a residential area is 2%. How many years it takes to double the population? (6 p.)

Model response: Let V be the original population. The population after n years is then $V \cdot 1,02^n$. Hence we get equation

$$V \cdot 1,02^n = 2V \Leftrightarrow 1,02^n = 2 \Leftrightarrow n \ln 1,02 = \ln 2 \Leftrightarrow n = \frac{\ln 2}{\ln 1,02} = 35.0028 \dots$$

Question 3. A bridge over a 96 m wide river has a profile of a circle arc (figure). The highest point of the bridge in the middle of river is 16 m higher than the ends of the bridge. What is the length of the bridge arc? (6 p.)



Model response: To find the arc length we need to find the radius of the circle and the central angle corresponding to the arc. We get

$$(r - 16)^2 + 48^2 = r^2 \Leftrightarrow r^2 - 32r + 16^2 + 48^2 = r^2 \Leftrightarrow r = 80$$

Half of the central angle corresponding to the arc is given by $\tan \alpha = \frac{48}{64} = \frac{3}{4}$, hence $\alpha = 0.643501 \dots$ (rad). Thus the central angle is $2\alpha = 1.2870 \dots$. Hence the length of the bridge arc is $1.2870 \cdot 80 = 102.96 \dots$ (meters).

Question 4. A javelin throw reached a length of 80 meters. Assume that the arc of the throw follows strictly a parabola in the xy -coordinate system in interval $x \in [0, 80]$. Determine the equation of the arc, when the starting point of the throw is $(0, 0)$, and the maximum height of is exactly 16 m. (6 p.)

Model response: Equation of a parabola is of form $y = ax^2 + bx + c$. We know that $y = 0$ if $x = 0$ or $x = 80$. Thus we obtain equations $0 = c$ ja $0 = a \cdot 80^2 + b \cdot 80 + c$. The top of the parabola is in the halfway of zeros 0 and 80, so $y = 16$ if $x = 40$. Hence we get equation $16 = a \cdot 40^2 + b \cdot 40 + c$. Because $c = 0$, we get a pair of equations

$$\begin{cases} 6400a + 80b = 0 \\ 1600a + 40b = 16 \end{cases}$$

The first equation gives $b = -80a$, and substituting this to the second one we get $-1600a = 16$, whence $a = -\frac{1}{100}$, and therefore $b = \frac{4}{5}$. Hence the requested parabola equation is $y = -\frac{1}{100}x^2 + \frac{4}{5}x$.